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This might have been done by shifting the latitude lines the proper amount southward on the map. The effect of refraction in longitude is inappreciable. Finally, it may be of service to add that the local times of the eclipse vary from midnight, or a few minutes after it, at the extreme eastern end of the eclipse region, to 9.41 p.m. at the western end. The longest duration of the eclipse anywhere is 17 minutes. This occurs in latitude 65° 59′ and in longitude 32° 48′ on the curve *Eclipse ends at Sunset*. The maximum obscuration is 0·011 of the sun's diameter, and is visible at the point GE, with the sun in the horizon. The maximum altitude of the sun within the eclipse region is less than seven and a half degrees.

The exceptional peculiarities of the present eclipse are the following:—

1. The moon's penumbra is barely more than tangent to the earth's disk.
2. The date is only a few days after the winter solstice, so that the penumbra falls upon the Antarctic Circle beyond the South Pole.
3. As a consequence of the fact last mentioned, the penumbra moves against the diurnal rotation, instead of with it as it does in other eclipses.
4. The eclipse occurs for some places at local midnight, so that there is an eclipse of the midnight sun.
5. This midnight sun eclipsed is the Christmas midnight sun.

Inequalities in the Period of the Eclipsing-Variable RT Persei.

By R. S. Dugan, Ph.D. (Plate 22.)

(*Communicated by H. N. Russell.*)

The variability of the star B.D. + 46°·740 was detected in 1904 by Mme. Ceraski, who found the star faint on a plate taken by Blajko in 1900. Observations of this star have since been made by Blajko, Graff, Tikhoff, Wendell, and Dugan.

During the years 1905-8 the writer made a series* of 904 observations, distributed thickly over the entire light variation and embracing many uninterrupted runs through primary and secondary minima. Each observation is the combined result of sixteen readings with the Pickering polarising photometer.

Blajko and Graff found the period to be $20^h\ 23^m\ 11^s$, Tikhoff found a period shorter by one second, while my discussion gave the period $20^h\ 23^m\ 10^s{\cdot}08$, with a very satisfactory representation of all available material.† Two years later I observed through three primary minima, and was much surprised to find them about

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* *Contributions from the Princeton Univ. Obs.*, No. 1.

† *Loc. cit.*, p. 15.

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seven minutes ahead of the predictions.* This has gone from bad to worse, and at present—about 3000 periods after the zero epoch in 1907—the residual O – C is about – 40 minutes.

The Harvard photographic history of this star, which goes back 7700 periods to November 1889, was generously sent me by Professor Pickering. The earliest photographs show a residual of about + 100 minutes. The average period over the entire known stretch of 10800 periods is therefore nearly a whole second shorter than the shortest of the periods found by the various observers. A satisfactory representation by this shorter period of the observations made from 1906 to 1908 is quite impossible. The minima run off ten minutes in the interval of 700 periods.

The material available for the discussion of the period is collected in Table I. and weighted with great care.

In the record of the Harvard photographs sent me there is given the time of the middle of the exposure to the third decimal of a day and the magnitude usually to a tenth, as estimated by Miss Leavitt. On many of the plates the star was "near edge," and in many cases there is a question mark after the magnitude. There are no multiple-exposure plates of RT Persei—each magnitude is an isolated observation. From a plot of all the observations, both photographic and visual, the average period was found. With this period the approximate shape of the photographic curve was found. The phase of each photographic observation, judged by the brightness of the star, was then found from this approximate photographic light-curve. After the final elements of light-variation were found, the mean photographic curve was again worked out and the times of minima revised. In some cases it was difficult to decide whether the observation was made on the descending or the ascending branch, and in a few cases the residuals are so large as to be of no value. It is clear that the time of eclipse is determined from an isolated photographic measure in this manner with far less accuracy than from a series of observations right through the entire eclipse, for which the shape of the mean light-curve is already known very precisely.

For lack of information, the weighting of the Blajko observations is an uncertain matter.†

The Graff observations are Argelander estimates, and have been published ‡ in detail. I have plotted the observations and weighted the results according to the accuracy with which a symmetrical curve, applied to the plot, apparently determines the time of minimum.

The Wendell observations, § made with an instrument similar to our own, are at least as good as my own. RT Persei was, however, but one among many stars on his observing list, and was seldom followed through more than one branch of the eclipse. I have found in my own work* with our photometer that the observations of one night are sometimes systematically higher than those

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* *Loc. cit.*, p. 16.

† *A.N.*, vol. clxvi. p. 155.

‡ *Hamburger Mitteilungen*, No. 8.

§ *Harvard Annals*, vol. lxix. pt. 2.

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of another night; that the mean curve of all minima usually fits the plot of each individual minimum quite closely, but for some nights it has to be pushed farther down to fit the points than on other nights. Since the determination of the time of mid-eclipse from observations made on one branch alone requires also a knowledge of the depth, the uncertainty of the determination is in such cases quite large.

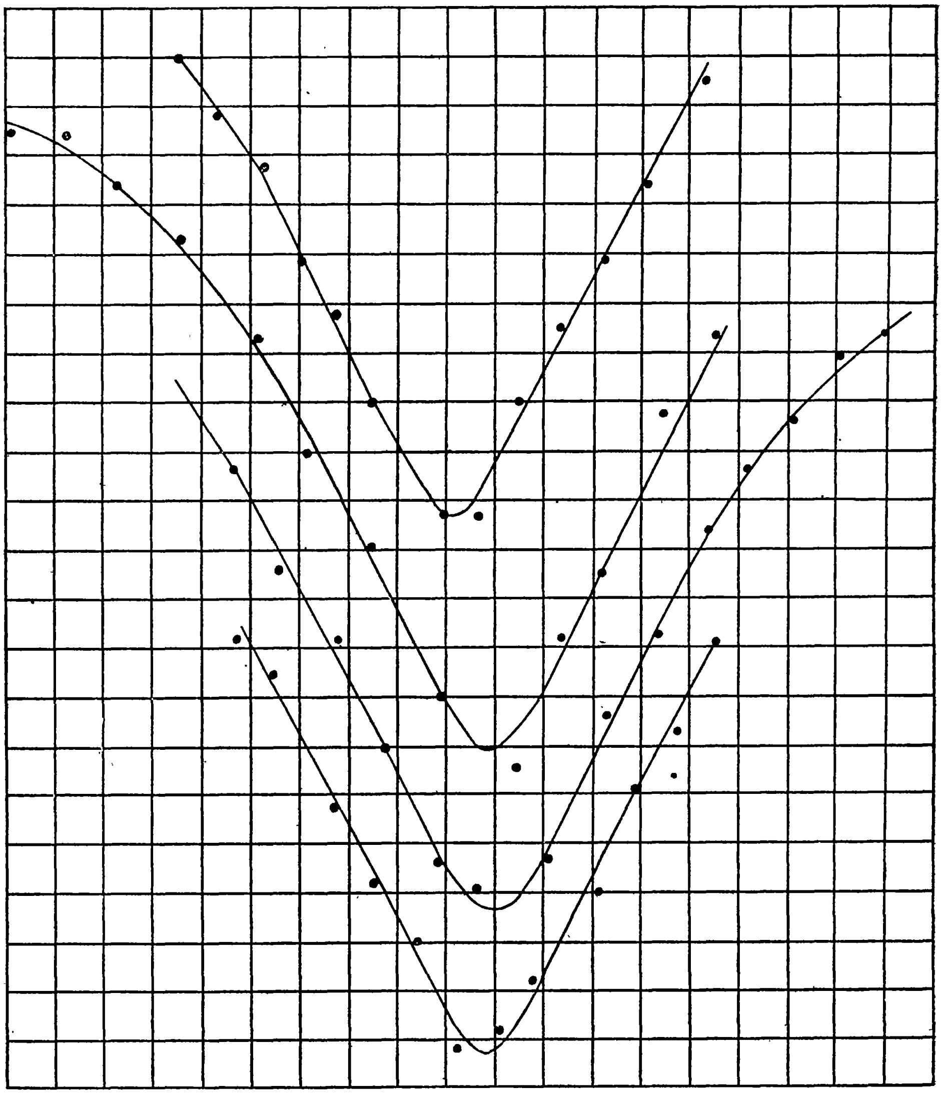

FIG. 1.

In order to give a convincing idea of how well the times of mid-eclipse are determined from photometric observations made rapidly throughout the eclipse, I have plotted the observations of four such minima in fig. 1. The mean curve is drawn on transparent paper and moved about until it seems to fit all the observations as well as possible. The distance between the co-ordinate lines is one-tenth magnitude vertically, and ten minutes horizontally. It

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is apparent that in these cases the mean curve could not be shifted bodily more than a minute or two without ceasing to represent the observations closely. The four minima have been displaced vertically and horizontally in the diagram, so that the points will not be confused.

Minimum + 558 was observed through a blue colour screen which I made for the purpose. Minimum + 2736 was observed photographically with the 6-inch Neilson camera.

In Table I. the second column contains the Greenwich Heliocentric Mean Time of minimum estimated from the observations made during the minimum. The Epoch E and the calculated time of eclipse C′ refer to the elements

$$\text{Min. } 1907 \text{ Oct. } 12^{d},\ 15^{h}\ 8^{m}\ 34^{s} \text{ Gr. H.M.T.} + 20^{h}\ 23^{m}\ 10^{s}{\cdot}08\text{E}$$
$$= 2417861{\cdot}63095 \text{ J.D.} + 0^{d}{\cdot}8494222\text{E.*}$$

The abbreviations are self-explanatory.

TABLE I.

J.D.	O. Gr. H.M.T. h m	E.	O – C′. m	Wt.	Source.	Remarks.
1317	18 4	– 7704	+ 101·5	2	H. Pg.	Edge of plate.
1685	13 17·5	7271	+ 103	½	,,	“Near edge. Magn. < 11·4.”
2085	14 56·5	6800	+ 90·5	3	,,	
2457	15 55	6361	+ 81	3	,,	
4325	12 27·5	4163	+ 47	3	,,	
5041	13 37	3320	+ 26	3	,,	
5086	14 13	3267	+ 34	1	,,	Desc. or asc. ? Average.
5239	10 55	3087	– 14	?	Bl. Pg.	Exposed, $1^{h}\ 14^{m}$.
5368	14 13	2935	+ 22	3	H. Pg.	
5487	12 0	2795	+ 6	1	,,	Near edge.
5615	18 4·5	2644	– 8	1	,,	,, ,,
5666	17 53	2584	+ 30	1	,,	? Poor.
5723	15 20	2517	+ 5	½	,,	? Edge, very poor.
5746	14 15	2490	+ 34	3	,,	
5751	16 14	2484	+ 14	3	,,	
6169	14 6	1992	+ 8	3	,,	
6175	12 59·5	1985	+ 19	3	,,	

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* *Loc. cit.*, p. 15.

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TABLE I.—*continued.*

J.D.	O. Gr.H.M.T. h	m	E.	O – C′. m	Wt.	Source.	Remarks.
6376	20	34	1748	+ 23	1	H. Pg	? Elongated images.
6427	19	23	1688	+ 2	3	,,	
6444	18	27	1668	– 38	?	,,	
6467	17	30	1641	0	2	,,	
6519	13	4	1580	+ 1	2	,,	Near edge.
6729	8	27	1333	+ 1	1	Bl.	
6734	10	42	1327	– 3	1	,,	
6741	5	45	1319	– 5	1	,,	
6755	16	16	1302	+ 8	½	H. Pg.	? Near edge, very poor.
6757	9	14	1300	0	3	Gr.	
6790	12	11	1261	– 3	5	Wen.	5 obs. on asc. br.
6903	11	35	1128	0	½	Gr.	
6904	8	1	1127	+ 3	5	,,	
6921	7	45	1107	+ 1	5	,,	
6926	10	10	1101	+ 7	5	,,	
6908	14	2	1122	– 1	3	Wen.	4 obs., all asc.
7177	20	16·0	805	– 2·5	15	D.	10 obs., both br.
7183	18	56·5	798	– 4	5	,,	9 obs., 2 on desc. br.
7229	15	50	744	– 2	10	Wen.	7 obs., 4 on asc. br.
7235	14	30	737	– 5	5	,,	5 obs., all asc.
7230	12	12·7	743	– 2	7	D.	7 obs., both br.
7235	14	34·9	737	+ 1	15	,,	14 obs., both br.
7240	16	52·5	731	– 0·5	15	,,	13 obs., both br.
7257	16	34·9	711	– 1	15	,,	10 obs., both br.
7258	13	0·5	710	+ 1	15	,,	15 obs., both br.
7264	11	45·0	703	+ 3·5	5	,,	7 obs., all asc.
7274	16	19·0	691	– 0·5	3	,,	6 obs., high on desc. br.
7292	12	28·0	670	+ 2	5	,,	10 obs., all asc.
7641	15	5	259	– 3	5	Wen.	5 obs., all desc.
7849	17	45·0	– 14	+ 1	10	D.	16 obs., large resid.
7850	14	7·0	– 13	– 0·5	5	,,	13 obs., 2 on desc. br.

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TABLE I.—*continued.*

J.D.	O. Gr. H.M.T. h m	E.	O−C′. m	Wt.	Source.	Remarks.
7854	20 3·3	−8	0	12	D.	11 obs., scattered.
7861	15 8·0	0	−0·5	15	,,	19 obs., both br.
7861	15 9	0	0	10	Wen.	7 obs., 4 desc.
7873	12 27	+14	−6	3	,,	4 obs., high on asc. br.
7907	11 59·5	+54	0	6	D.	14 obs., mostly asc.
7916	20 14·0	+65	−0·5	15	,,	14 obs., both br.
7917	16 37·3	66	−0·5	15	,,	18 obs., both br.
7918	13 2·0	67	+1	15	,,	13 obs., both br.
7922	18 56·5	72	0	7	,,	10 obs., 2 asc.
8335	14 34	558	−2	10	D. Blue	17 obs., both br.
8589	13 57	857	−7	10	D.	8 obs., 3 desc.
8589	13 50	857	−13·5	7	Wen.	6 obs., all asc.
8600	15 0	870	−4·5	4	,,	5 obs., 3 asc.
8600	14 57·5	870	−7	15	D.	14 obs., both br.
8707	15 28	996	−16	3	Wen.	3 obs., all asc.
8713	14 17	1003	−9	7	,,	6 obs., 3 on each br.
8713	14 18	1003	−8	15	D.	17 obs., both br.
9102	15 4	1461	−13	15	,,	14 obs., both br.
0185	14 50·5	2736	−46	5	D. Pg.	5 exp., 20 min. each, 1 desc.
0457	10 27	3056	−43	5	D.	4 obs., high on asc. br.
0484	14 50	+3088	−41	5	,,	7 obs., large resid.

The normals, whose grouping is indicated in Table I. by spacing, are given in Table II.

TABLE II.

E.	O−C′. m	Wt.	Source.	O−C″. m	O−C. m
−7500	+102	2	H. Pg.	+6	−5
6580	+86	6	,,	+3	+1
4163	+47	3	,,	−3	+6

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TABLE II.—*continued.*

E.	O − C′.	Wt.	Source.	O − C″.	O − C.
	m			m	m
3300	+ 28	4	H. Pg.	− 10	− 3
3087	− 14	?	Bl. Pg.	− 49	—
2935	+ 22	3	H. Pg.	− 11	− 2·5
2720	− 1	2	,,	− 32	—
2500	+ 24	7	,,	− 4	+ 6
1988	+ 14	6	,,	− 6	+ 6
1658	+ 4	8	,,	− 12	− 0·5
1315	− 2	3	Bl.	− 13	− 3
1302	+ 8	½	H. Pg.	− 3	+ 7
1300	0	3	Gr.	− 11	− 1·5
1261	− 3	5	Wen.	− 13	− 3·5
1107	+ 3	15	Gr.	− 5	+ 3
1122	− 1	3	Wen.	− 9	− 1
804	− 3	20	D.	− 7	− 3
742	− 3	20	Wen.	− 6	− 3
739	0	22	D.	− 3	0
731	− 0·5	15	,,	− 3·5	− 0·5
711	− 1	15	,,	− 3	− 1
710	+ 1	15	,,	− 2	− 1
687	+ 1·5	13	,,	− 0·5	+ 2·5
259	− 3	5	Wen.	0	− 3·5
− 14	+ 0·5	15	D.	+ 7·5	+ 1
− 8	0	12	,,	+ 7	+ 0·5
0	− 0·5	15	,,	+ 6·5	0
+ 2	− 1·5	13	Wen.	+ 5·5	− 1
+ 60	− 0·5	21	D.	+ 7·5	0
66	− 0·5	15	,,	+ 7·5	0
69	+ 0·5	22	,,	+ 8·5	+ 1
558	− 2	10	D. Blue	+ 12	+ 0·5
857	− 7	10	D.	+ 12	0
860	− 11	11	Wen.	+ 8	− 4
870	− 7	15	D.	+ 12	0
1000	− 11	10	Wen.	+ 10	− 2
1003	− 8	15	D.	+ 13	+ 1
1461	− 13	15	,,	+ 14	+ 3·5
2736	− 46	5	D. Pg.	− 2	− 9
+ 3072	− 42	10	D.	+ 7	− 1·5

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A linear correction to the epoch and period was found graphically from the residuals O – C′. The new average period is $0^{s}\cdot82$ shorter than the old, and leaves the new residuals, O – C″, given in column 5. These residuals are plotted in fig. 2, and the source of each designated by a letter. In considering this plot one should also bear carefully in mind the weights given in Table II.

To explain the steep ascent from – 1400E to + 400E and the maintaining of this high level to + 3000E it is necessary to consider two inequalities. The curve in fig. 2 is the combination of two sine terms, one with a coefficent of 12 minutes and a period of 11880E or 27½ years, the other with a coefficient of 5 minutes and a period one-third as long as the other, or 3960E. The two terms go through their zero values together at – 500E. Column 6 of Table II. gives the residuals O – C from this curve. The values found for these two inequalities are apparently near the truth.

A question of great interest now is whether the secondary minimum shifts its position in either of these periods. Comparatively few observations have been made during secondary minimum. What material there is has been collected in Table III. C′, C″ are the times of the points midway between primary

TABLE III.

J.D.	O. Gr. H.M.T.	E.	O – C m	O – C″. m	Wt.	Source.
4969	18 53	– 3405	+ 19	– 20·5	1	H.Pg.
4998	16 27	3371	+ 46	+ 7	1	,,
6499	14 6	1604	+ 7	– 8	1	,,
6787	12 53	1265	0	– 10·5	3	Wen.
6792	15 7	1259	– 5	– 15·5	3	,,
6793	11 27	1258	– 8	– 18·5	3	,,
7180	19 53	798	+ 13·5	+ 9·5	5	D.
7220	18 12	755	+ 22·5	+ 19·5	3	,,
7469	14 53	462	– 3·5	– 3	5	Wen.
7857	19 15	– 5	– 9	– 2	15	D.
7858	15 40	– 4	– 7·5	– 0·5	8	,,
7862	21 32	+ 1	– 11	– 4	8	,,
7863	18 7	+ 2	0	+ 7	15	,,
7864	14 10	+ 3	– 19·5	– 12·5	15	,,
0453	15 0	+ 3051	– 25·5	+ 23	5	,,
0459	13 26	+ 3058	– 41·5	+ 7	5	,,

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minima calculated by the old and the new elements respectively. Removing now the effect of the short inequality—column 4 of Table IV.,—that is, assuming that the position of secondary with reference to the primary does not vary with the short inequality, we obtain residuals R_1, while the residuals R_2 show the effect of the short inequality alone.

TABLE IV.

E.	O − C″.	Short.	R_1.	Long.	R_2.	Wt.
	m	m	m	m	m	
−3388	− 7	−5	−12	+12	+ 5	2
−1604	− 8	+4·5	− 3·5	+ 7	− 1	1
−1260	−15	+4·5	−10·5	+ 5	−10	9
− 780	+13	+2·5	+10·5	+1·5	+14·5	6
− 462	− 3	+0·5	− 2·5	0	− 3	5
0	− 2·4	−3	− 5·4	− 3	− 5·5	27
0	− 2·8	−3	− 5·8	− 3	− 5·8	30
+3056	+15	+4	+19	− 7·5	+ 4	10

If the shorter inequality is caused by a revolution of the line of apsides, the time of secondary minimum should shift back and forth through a range of 20 minutes. The residuals R_2 should follow the curve drawn out in fig. 3. The weight of the point is given in the figure as well as a letter indicating the source. In considering this figure it should be remembered that the entire drop in secondary minimum is little over a tenth of a magnitude, and would not show at all in any but very careful observations. The photographic estimates to a tenth of a magnitude are of little value in fixing the time of secondary eclipse. My own observations follow the curve approximately. They are at least in the same sense as the curve. I have plotted in fig. 4 the observations made through secondary minimum $+2\frac{1}{2}$ E, which is one of two combined in the point of weight 30. The vertical-magnitude-scale is double that used in fig. 1. The time of secondary minimum is determined with much less accuracy than that of primary minimum. The Wendell point at −1260 E is far from the curve. The observations of the three minima which are combined into this point are plotted in fig. 5. In no case were the observations continued right through the minimum. The determination of the time of minimum is in such cases quite uncertain. Furthermore, it is noticeable in fig. 2 that for some reason the Wendell points nearly all lie below the curve.

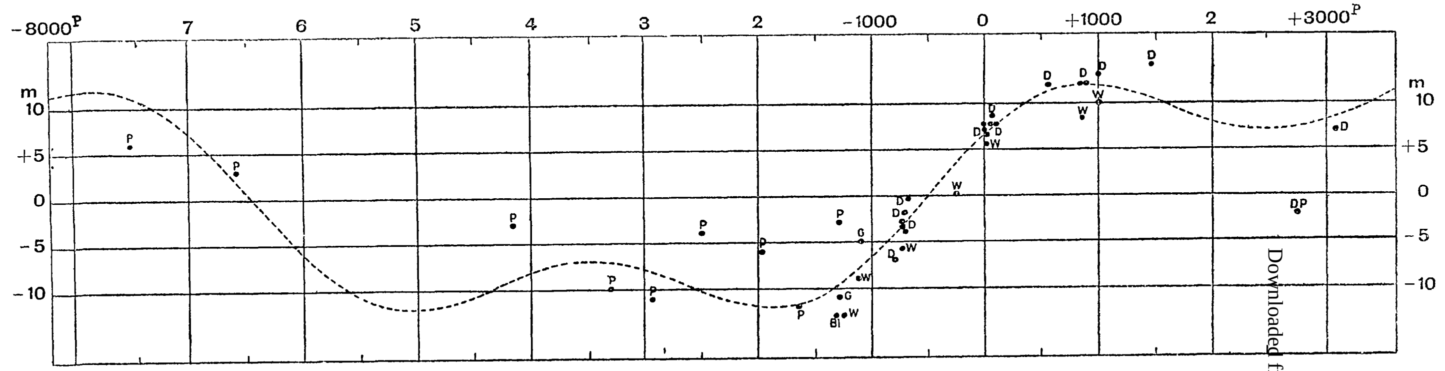

Fig. 2.

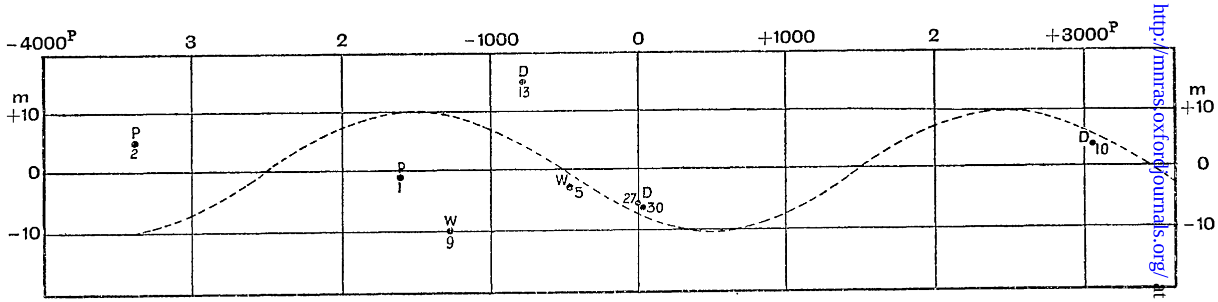

Fig. 3.

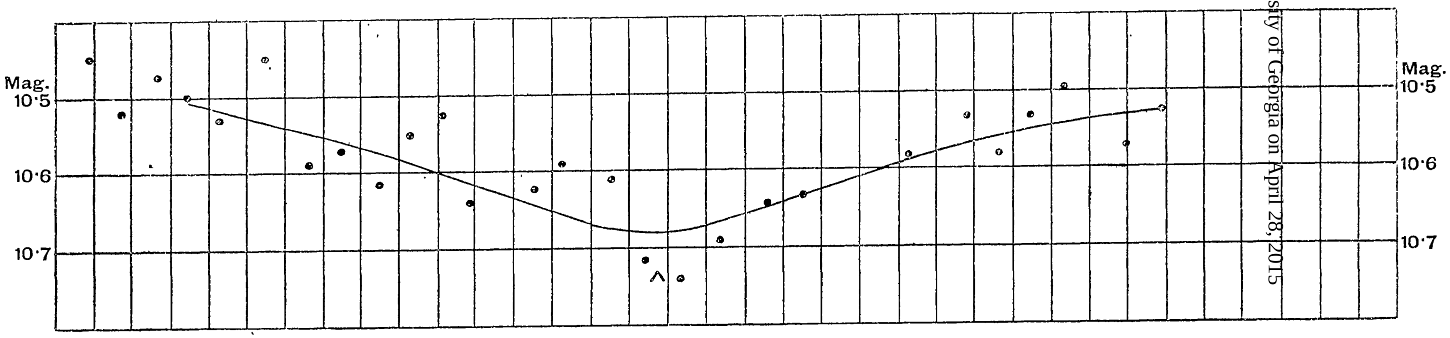

Fig. 4.

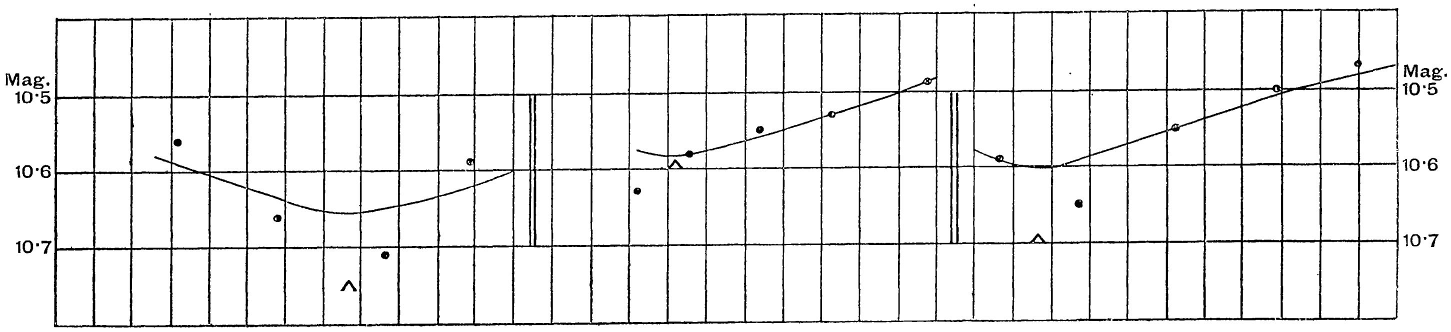

Fig. 5.

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The last of the residuals R_1, at $+3056$ E, makes it improbable that the line of apsides is revolving in the period of the longer inequality.

From the discussion of the observations made in 1905–8 it was found that the stars were prolate.* There is probably some flattening at the poles which is beyond the reach of the observations. The distribution of mass in the stars is also unknown. The prolateness, or ellipticity of the disc when at its maximum area, was found to be 0·02. This amount of prolateness in stars of equal mass and homogeneous would cause the line of apsides to revolve in about the period of the shorter inequality.

The amplitude of the shorter inequality corresponds to an orbital eccentricity of 0·013, and the longitude of periastron, counting from the node in the direction of motion, was apparently about 134° in October 1907. Periastron was in the line of sight about a year earlier. The mean curve in 1907 shows a displacement of the secondary toward the preceding primary, and with less distinctness a slightly longer secondary than primary eclipse. This would be the case if the line of apsides was directed to the line of sight as found from the other evidence. The shallowness of the secondary eclipse prevents an accurate estimate of the duration of the eclipse.

SUMMARY.

The residuals of the primary minima plotted in fig. 2 prove that the average period is shorter than hitherto supposed, and that there exist at least two inequalities in the period. The residuals of the secondary minimum plotted in fig. 3 and the mean curve of the observations made chiefly in 1907 indicate a revolution of the line of apsides in the period of the shorter inequality, caused by the ellipticity of the stars.

One or two minima, both primary and secondary, carefully observed each year for the next ten years, should throw much light upon this very important question.

The new elements are

$$
\begin{aligned}
&1907\ \text{Oct. } 12^{d},\ 15^{h}\ 1^{m}\!\cdot\!6\ \text{Gr. H.M.T.} + 20^{h}\ 23^{m}\ 9^{s}\!\cdot\!26E \\
&\qquad + 12^{m} \sin(0^{\circ}\!\cdot\!030E + 15^{\circ}) + 5^{m} \sin(0^{\circ}\!\cdot\!091E + 45^{\circ}) \\
&= 2417861\!\cdot\!6261\ \text{J.D.} + 0^{d}\!\cdot\!8494127E + 0^{d}\!\cdot\!0083 \sin(0^{\circ}\!\cdot\!030E + 15^{\circ}) \\
&\qquad + 0^{d}\!\cdot\!0035 \sin(0^{\circ}\!\cdot\!091E + 45^{\circ})
\end{aligned}
$$

and the new period

$$
\begin{aligned}
P = 20^{h}\ 23^{m}\ 9^{s}\!\cdot\!26 &+ 0^{s}\!\cdot\!38 \cos(0^{\circ}\!\cdot\!030E + 15^{\circ}) \\
&+ 0^{s}\!\cdot\!47 \cos(0^{\circ}\!\cdot\!091E + 45^{\circ}).
\end{aligned}
$$

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* *Loc. cit.*, p. 39.

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The ephemeris of primary minima for 1915 in Gr. H.M.T. is as follows:—

Epoch.		d	h	m
+3105	Jan.	1	1	24·3
3141	Feb.	0	15	17·9
3174	Mar.	0	16	2·1
3210	Apr.	0	5	55·9
3246	May	0	19	49·6
3282	June	0	9	43·4
3318	July	0	23	37·1
3354	Aug.	0	13	30·8
3390	Sept.	0	3	24·5
3426	Oct.	0	17	18·3
3462	Nov.	0	7	12·0
3498	Dec.	0	21	5·9
	1916.			
+3534	Jan.	0	10	59·6

Princeton University Observatory :
1915 *May* 20.

Inequalities in the Period of the Eclipsing-Variable Z Draconis.

By R. S. Dugan, Ph.D.

(*Communicated by H. N. Russell.*)

This star is B.D. + 73°·533, and was found to be a variable by Mme. Ceraski by comparison of plates taken by Blajko.* It appears on a great many plates taken at Harvard, the earliest minimum having been recorded in 1890. The Harvard material has been kindly placed at my disposal in manuscript. It has been utilised in the same manner as the observations of RT Persei.† Some of these observations were rejected on account of their large residuals. Several minima were observed visually, in 1903, by Blajko,‡ and the estimated times published. Miss Whitney observed one minimum in 1903 with a Rumford photometer.§ Reed and Daniel observed seven minima in 1903 with a Rumford photometer attached to the 23-inch Princeton telescope.|| Their observations are very numerous, but the artificial star seems to have varied in intensity, resulting in a variable shape of the curve. Nijland has published ¶ the times of nineteen minima observed by

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* *A.N.*, vol. clxi. p. 159.

† *M.N.*, vol. lxxv. p. 692.

‡ *A.N.*, vol. clxii. p. 383.

§ *Pop. Ast.*, vol. xi. p. 428.

|| *A.J.*, vol. xxiii. p. 188.

¶ *A.N.*, vol. clxxvi. p. 169.